

## COSMOSEMS for Magnetostatic Analysis

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This article is intended as a brief evaluation of results obtained with COSMOSEMS, comparing those results with closed-form analytic solutions. We shall refer to COSMOSEMS results as finite element analysis (“FEA”) results, while we describe calculations performed using analytic expressions as “analytic solutions.” The solver employed was an FFE solver and meshes were performed with a “draft” quality tetrahedral mesh geometry.

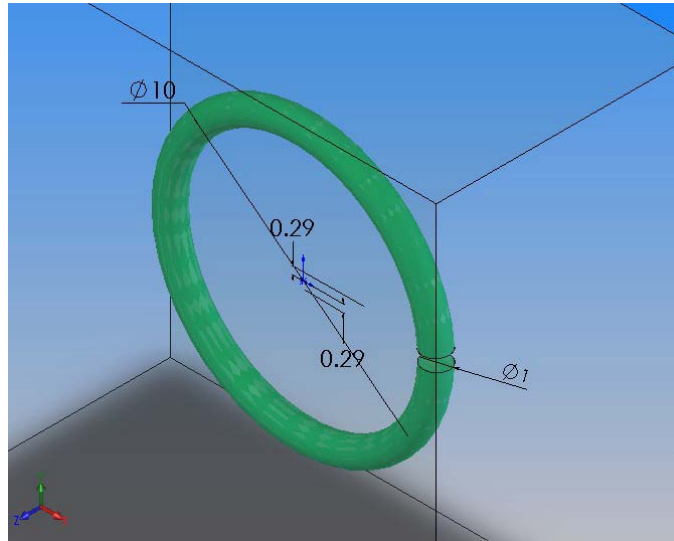
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### Simple Coil in Air

We explore a relatively straightforward, geometrically simple, closed-form, quantitative problem on p. 227 of Purcell’s 1985 Second Edition of “Electricity and Magnetism.”<sup>1</sup> In this example a simple wire coil of radius  $R$  (in cm), is driven by a current  $I$  (in esu/sec), and the resulting magnetic field in the ‘z’- axis direction (“ $B_z$ ”, in Gauss), at the center of the coil may be described as:

$$B_z = \frac{2\pi(I)}{cR} , \quad (1)$$

where  $c$  is the speed of light in vacuum ( $\sim 3 \times 10^{10}$  cm/sec). The model tested against this relation is shown in Figure 1. The ring sits in the ‘x-y’ plane,



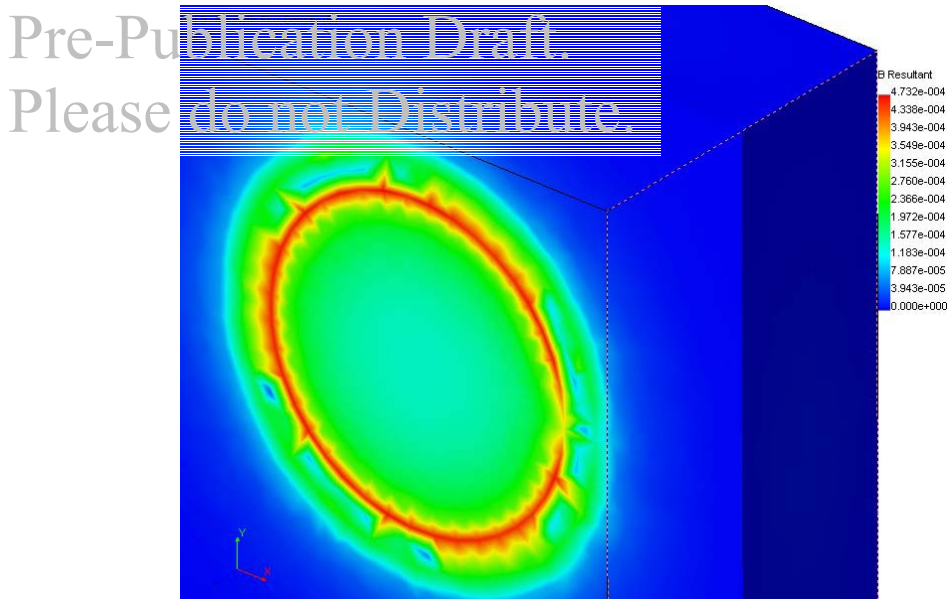
**Figure 1. A simple coil configuration used to quantitatively test COSMOSEMS when calculating the  $B$  field. The units are mm, so the coil radius is 0.5 cm, while the volume of surrounding air is bounded by a prism 2 cm on a side by 1 cm deep.**

<sup>1</sup> McGraw-Hill, Inc., Edward M. Purcell, Vol. 2 of Berkeley Physics Course, Equation 6-42, Figure 6.15.

We can express the  $B$  field calculated with Equation 1 for this arrangement, converted to Tesla, as

$$B_z = \frac{(6.28)(1\text{amp})(3 \times 10^9 \frac{\text{esu}}{\text{sec-amp}})}{(3 \times 10^{10} \frac{\text{cm}}{\text{sec}})(0.5\text{cm})} (10^{-4} \frac{\text{Tesla}}{\text{Gauss}}) = 1.26 \times 10^{-4} \text{Tesla} . \quad (2)$$

For the FEA test, we set one amp flowing through the coil. The coil material was defined as copper and its ends were left truncated instead of pigtailed to simplify the analysis. The “airbox” dimensions were chosen to exceed the COSMOS criterion that each linear dimension should be 1.5 times the associated model dimension. In this case, the length and width of the airbox were set to 20 mm, while the depth was set to 10 mm. The “airbox” faces were set to the transverse flux boundary condition, and the mesh dimensions were customized to provide as much detail as possible. We checked the results for the resultant  $B$  field vs. the  $B_z$  field and found effectively no difference, indicating that the field near the ‘z’- axis is overwhelmingly directed along the ‘z’- axis.

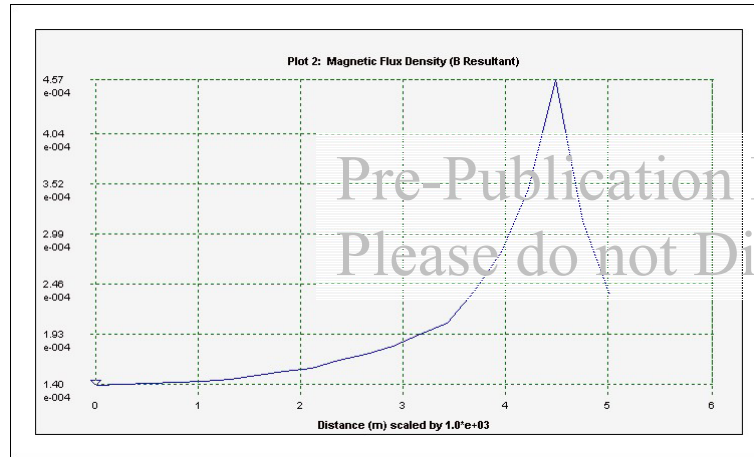


**Figure 2. The  $B$  field magnitude for the single coil of Figure 1, with 1 Amp ( $3 \times 10^9$  esu/sec) flowing. The magnitude is reported by COSMOSEMS in Tesla ( $1 \text{ Tesla} = 10^4 \text{ Gauss}$ ). The field in the blue-green-colored region is around  $1.4 \times 10^{-4}$  Tesla.**

COSMOSEMS allows 2D plots to be derived from the volume data by prompting the operator for start- and end-points for a line through the volume. Using this technique, we obtained the plot in Figure 3 for the  $Z = 0$  plane, with the path from the origin ( $Y=0$ ) along the ‘y’- axis to the point  $Y=5$ . The FEA result at  $Y = 0$  is  $1.40 \times 10^{-4}$  Tesla, about 11% higher than the solution in Equation 2. This difference could be due to the rather

large wire diameter (1 mm) chosen, relative to the ring diameter (10 mm), coupled with the fact that the ring is not truly complete (there is a 0.58 mm or  $\sim 6.7^\circ$  gap).

Although the range of values appears to increase dramatically from the origin to the current ring, the B value actually only varies from  $1.40 \times 10^{-4}$  at the origin to  $1.67 \times 10^{-4}$  Tesla at the  $y = 2.5$  mm position.



**Figure 3. The field magnitude versus distance along the ‘y’-axis. At the coil center, the FEA solver-reported field value was about  $1.40 \times 10^{-4}$  Tesla, or about 11% high.**

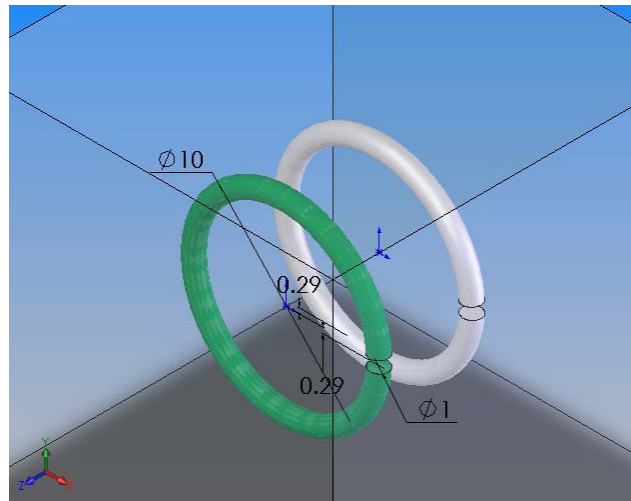
This quick evaluation indicates there are several issues to watch carefully when setting up and solving even simple models. As FEA simulations are employed to tackle more complex designs, these issues become more important. Critical issues include:

- Model geometries (relative size of conductors to airbox, e.g.) must be managed to allow prudent meshing
- Mesh control features that dictate high-resolution mesh regions should be employed as necessary (especially helpful for tiny conductors in large volumes)
- The “coil define” option allows the choice of stranded or solid conductors as a tool to emulate real configurations
- In hopelessly disparate relative dimension cases (hundreds of wraps of  $10 \mu\text{m}$  wire in a 1 m box, e.g.), a sheet or bar conductor may be employed, where the current is then set proportionately higher
- Attention to the boundary conditions at the airbox surface is important
- Simplified “sanity checks” should be performed on more complex models

### Helmholtz Coil Assembly

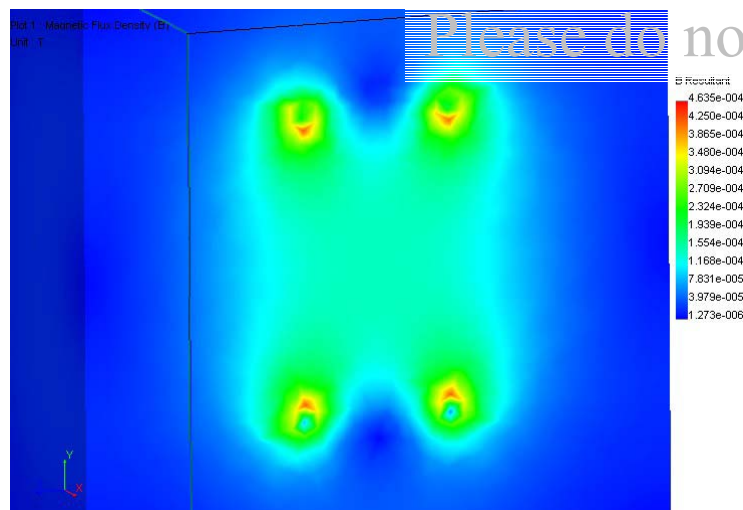
To investigate the FEA-predicted uniformity of a Helmholtz coil assembly, we employed the configuration shown in Figure 4, using two instances of the same conductor ring from the previous problem. The criterion for Helmholtz coil placement is to set the distance  $b$  between the two coils equal to the radius of each coil. So for this example, we spaced the

10 mm diameter coils 5 mm apart. The second order partial differential equation for the field halfway between the two coils (on the axis) vanishes under this condition.



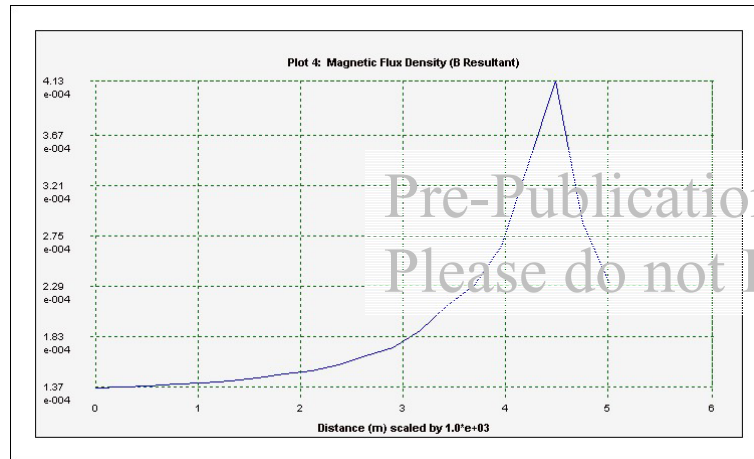
**Figure 4. The arrangement used to explore the field uniformity of Helmholtz coils. The “airbox” in this case is 20 mm deep.**

The FEA results are shown in Figure 5. Not only is the field quite uniform at the geometric center of the assembly, but it is quite uniform throughout much of the volume between the two coils.

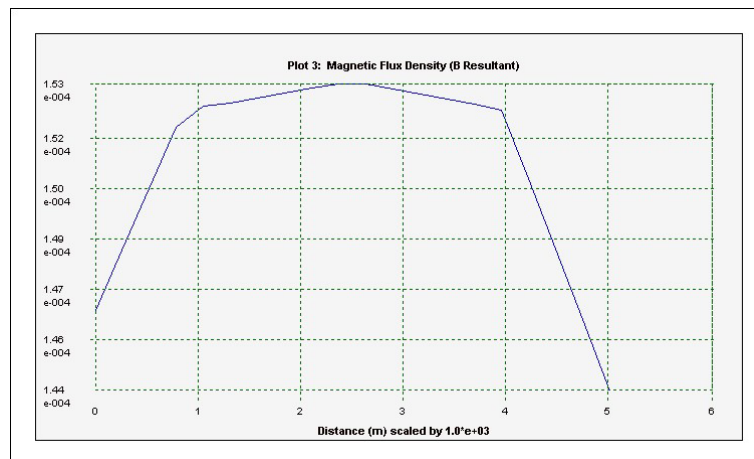


**Figure 5. This slightly oblique view of the resultant  $B$  field shows excellent field uniformity within the evaluation volume.**

Figure 6 is a plot along the 'y'-axis of the arrangement, while Figure 7 is a plot along the 'z'- axis.



**Figure 6. Plot of  $B$  field along the 'y'- axis, starting at the origin. The value ranges from  $1.37 \times 10^{-4}$  to  $1.83 \times 10^{-4}$  Tesla going from the origin to  $Y = 3$  mm.**



**Figure 7. Plot of  $B$  field along the 'z'- axis, starting at the origin. The value changes by only  $5 \times 10^{-7}$  Tesla ( $\sim 0.4\%$ ) over the central 3mm.**

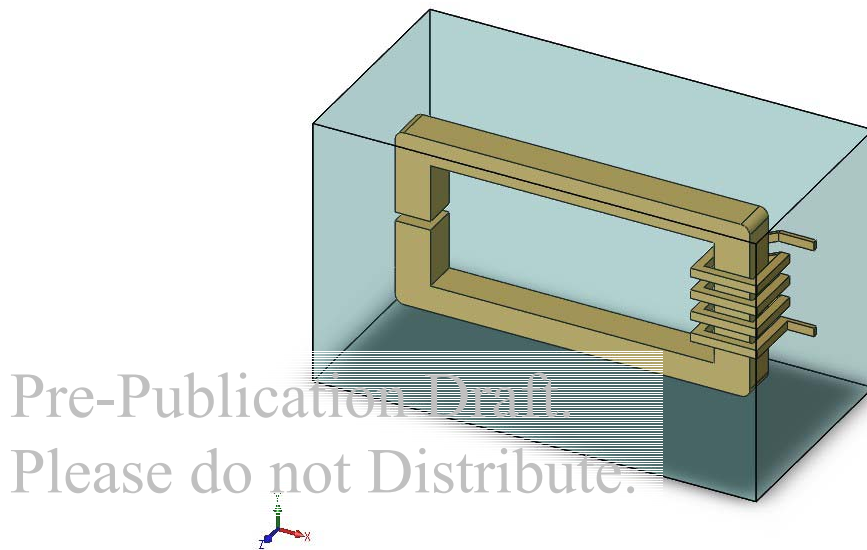
### Iron Core with Gap

As an interesting simulation, we have worked up an iron core with a profile of 20 mm by 40 mm, where the iron bar is 4 mm by 4 mm (see Figure 8). A square cross-section (1 mm x 1 mm) copper coil comprising 4 turns and spaced from the core by 0.75 mm per side is powered with a 1 amp current source. The entire assembly is surrounded by an airbox measuring 30 mm by 50 mm by 25 mm deep. A gap of 1 mm is situated on the arm opposite the excitation coil position.

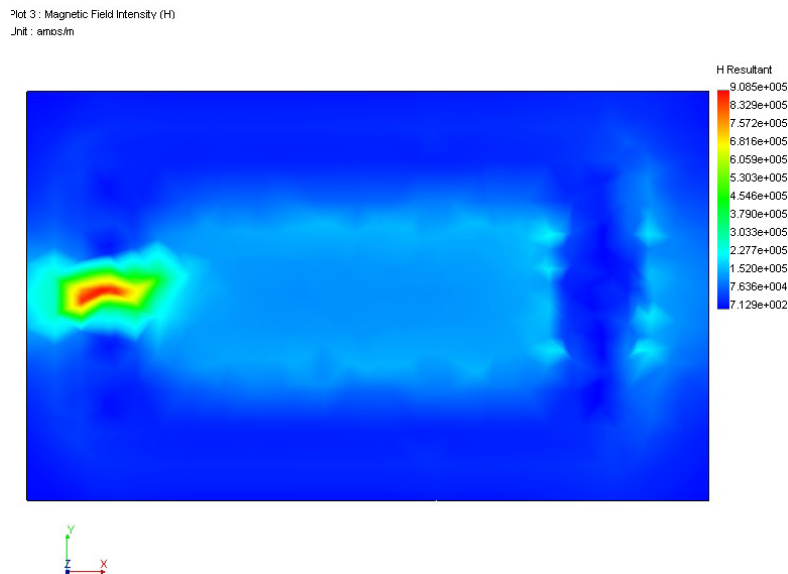
The excitation to the coil was 1 volt across the faces of the contact prongs. Figure 9 shows a view from the front of the assembly, where a cutting plane passes through the origin, and the  $H$  field resultant magnitude is represented. The fact that the  $H$  field outside the iron is the peak value makes sense, (in the CGS system)

$$H = B - 4\pi M \quad . \quad (3)$$

Therefore, the  $H$  field within the iron is much lower, due to the magnetization  $M$  of iron.



**Figure 8. The iron core and coil assembly, within the airbox.**



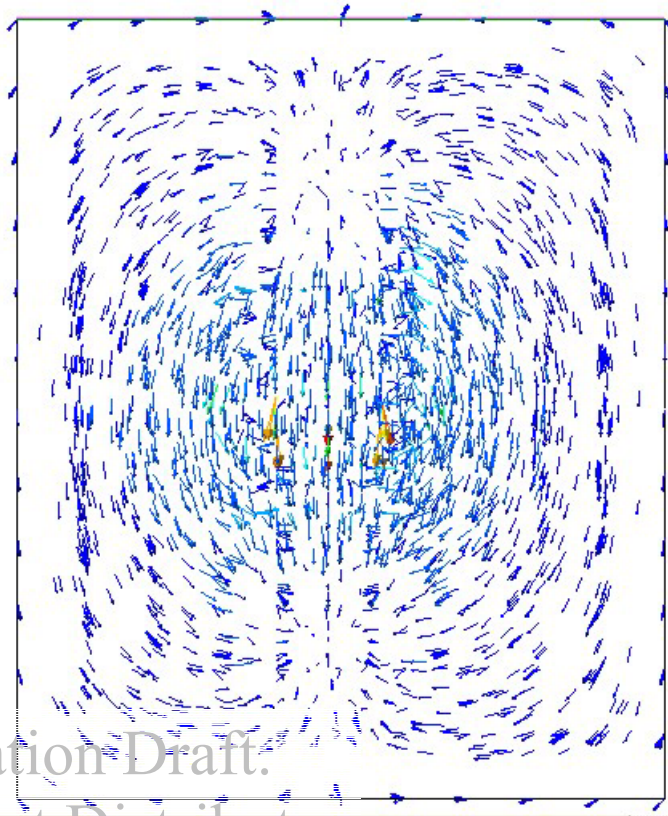
**Figure 9. The  $H$  field resultant magnitude for the iron core assembly in Figure 8. The peak value, which is in the gap, is reported as  $\sim 9 \times 10^5 \text{ Am}^{-1}$ .**



## Conclusion

Achieving accurate results with COSMOSEMS is a process that requires careful selection of mesh parameters and thoughtful definition of boundary conditions. It is wisest to compare results with analytic solutions (where possible) as guidance to growing a proper solution.

For qualitative analysis, the graphical representation of results provides useful insights into the nature and structure of the fields.



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